Demonstration of Controllable Temporal Distinguishability in a Three-Photon State

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Multi-photon interference is at the heart of the recently proposed linear optical quantum computing scheme[1] and plays an essential role in many protocols in quantum information[2, 3, 4]. Indistinguishability is what leads to the effect of quantum interference. Optical interferometers such as Michaelson interferometer provide a measure for second-order coherence at one-photon level[5] and Hong-Ou-Mandel interferometer[6] was widely employed to describe two-photon entanglement and indistinguishability[7, 8, 9]. However, there is not an effective way for a system of more than two photons. Recently, a new interferometric scheme [10, 11, 12] was proposed[13] to quantify the degree of multi-photon distinguishability. Here we report an experiment to implement the scheme for three-photon case. We are able to generate three photons with different degrees of temporal distinguishability and demonstrate how to characterize them by the visibility of three-photon interference. This method of quantitative description of multi-photon indistinguishability will have practical implications in the implementation of quantum information protocols.

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Early pioneers of quantum optics developed a complete theory of quantum coherence[14, 15], which was able to explain and predict a number of quantum phenomena of light such as photon anti-bunching, sub-Poissonian photon statistics, and squeezed state of light[16]. However, the theory was geared in close connection to the classical coherence theory with an emphasis on the wave aspect of light and is best to characterize coherence in the second-order of the field amplitudes. With the recent advent of quantum information science, most of the applications are photon-number based system, i.e., a system with a definite number of photons. Thus the Glauber's quantum coherence theory fell short to give a direct account of the quantum entanglement of a multi-photon system.

Quantum entanglement is best described by the quantum interference effect. As is generally believed, indistinguishability of photon's paths directly leads to the quantum interference effect. Early method[7, 8, 9] for characterizing the two-photon temporal distinguishability is by sending the fields into a Hong-Ou-Mandel two-photon interferometer[6] and measuring the visibility of the interference dip. A number of attempts[17, 18, 19] were made to characterize the temporal distinguishability for the two independent pairs of photons from parametric down-conversion and a quantity $\mathcal{E}/\mathcal{A}$ is identified[20, 21] to characterize the temporal distinguishability between different pairs of down-converted photons (photons within a pair are in the same temporal mode and are indistinguishable under certain condition). However, a generalization to arbitrary photon number is not possible until a new multi-photon interferometric scheme, the so-called "NOON" state projection measurement, was proposed[10] and realized[11, 12] recently. The new interferometric scheme was initially used to demonstrate the N-photon de Broglie wavelength[22, 23] without the need of a maximally entangled N-photon state (the so-called "NOON" state)[10, 11, 12]. But it is shown[13] that the visibility or the relative depth of the interference dip in the "NOON" state projection measurement can be used to quantitatively characterize the different scenarios of temporal distinguishability of a multi-photon system.

In this letter, we wish to report on an experimental procedure based on the "NOON" state projection measurement scheme to characterize the temporal distinguishability of a controllable three-photon system generated from two parametric down-converters. We observed a three-photon interference dip with 91% visibility when all three photons are indistinguishable and two dips with respective 45% and 39% visibility when two among the three photons become distinguishable. We compare the measured visibility with a model of pulse pumped parametric down-conversion and obtain good agreement.

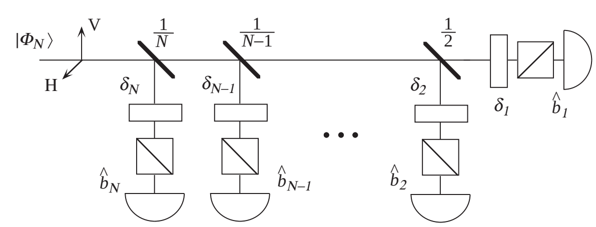

FIG. 1: *A NOON-state projection measurement for N photons*

We start by describing the "NOON" state projection measurement shown in Fig.1 for N-photon case. The incoming field of both horizontal(H) and vertical(V) polarizations is divided into N equal parts by N-1 beam splitters. Each part passes through a wave plate that introduces a relative phase shift of $\delta_k = 2\pi(k-1)/N(k = 1, ..., N)$ between the H and V polarizations. It is then

projected to the 135°-direction by a polarizer. The field operator at the kth detector is expressed via the input operators by

$$\hat{b}_k = (\hat{a}_H - \hat{a}_V e^{i\delta_k})/\sqrt{2N} + \hat{b}_{k0}, \qquad (k = 1, ..., N) \quad (1)$$

where $\hat{b}_{k0}$ is related to the vacuum input to the beam splitters. The N-photon detection rate is proportional to

$$P_N = \Big\langle \prod_{k=1}^{N} \hat{b}_k^\dagger \prod_{k=1}^{N} \hat{b}_k \Big\rangle, \quad (2)$$

where the average is over the quantum state $|\Phi_N\rangle = \sum c_k |k\rangle_k |N-k\rangle_V$ of $\hat{a}_{H,V}$. But because of the identity

$$\prod_{k=1}^{N} \hat{b}_k = (\hat{a}_H^N - \hat{a}_V^N)/(2N)^{N/2}, \quad (3)$$

Eq.(2) becomes

$$P_N = 2N!|\langle \Phi_N | NOON\rangle|^2/(2N)^N, \quad (4)$$

where $|NOON\rangle \equiv (|N\rangle_H|0\rangle_V - |0\rangle_H|N\rangle_V)/\sqrt{2}$ is the NOON-state[24, 25]. So the N-photon coincidence rate is proportional to the probability of projecting $|\Phi_N\rangle$ to the NOON state.

If the input state $|\Phi_N\rangle$ has both non-zero c_0 and c_N and experiences a relative phase shift of δ between H and V polarizations, it is easy to show that

$$P_N \propto |c_0|^2 + |c_N|^2 - 2|c_0 c_N| \cos(N\delta + \delta_0), \quad (5)$$

which demonstrates an N-photon de Broglie wave length. On the other hand, if the input state is orthogonal to the NOON state, in particular if $|\Phi_N\rangle = |k, N-k\rangle$ with $k \neq 0, N$, we will have zero coincidence for N-photon detection, i.e., $P_N = 0$, due to orthogonal projection. When we examine Eq.(3), we find this orthogonality stems from the absence of the terms of $\hat{a}_H^k \hat{a}_V^{n-k} (k \neq 0, N)$. A further examination shows that the disappearance of the coefficients of $\hat{a}_H^k \hat{a}_V^{n-k} (k \neq 0, N)$ is because of complete destructive N-photon interference[26].

Of course, the above analysis is based on a single mode description in which all N photons are in one indistinguishable temporal mode. In reality, the N photons may not be in a single temporal mode. Then we will not have complete destructive interference and P_N will be a non-zero value depending on the degree of temporal distinguishability. This is precisely the proposal by Ou[13] to use the visibility of the N-photon interference for quantitative characterization of temporal distinguishability of the $|k, N-k\rangle (k \neq 0, N)$ state. For the state of $|N-1\rangle_H |1\rangle_V$, in particular, we have the N-photon interference visibility $\mathcal{V}_N = m/(N-1)$ when m H-photons are in one temporal mode that is distinguishable from other $N-m-1$ H-photons (the V-photon must have the same temporal mode as the m H-photons)[13].

To demonstrate experimentally the procedure for characterizing N-photon temporal distinguishability, we need to prepare a state of the form $|N-1\rangle_H|1\rangle_V$ that is orthogonal to the NOON state. Note that this state is fundamentally different from the states used in Refs.[11, 12] where a non-orthogonal state to the NOON state must be employed for the demonstration of N-photon de Broglie wavelength. We are able to generate a three-photon state in the form of $|2_H, 1_V\rangle$ with controllable temporal distinguishability by two type-II parametric down-converters shown in Fig.2. Two BBO crystals are pumped synchronously by two pulses of 150 fs length from a frequency doubled Ti:sapphire laser operating at 780 nm. The H-photon from BBO1 is injected into the H-polarization mode of BBO2 whereas the V-photon of BBO1 is detected to produce a trigger for three-photon coincidence. The H-photon from BBO2 together with the H-photon from BBO1 are coupled into a single mode fiber and then are combined by a polarization beam splitter (PBS) with the V-photon from BBO2 via another single mode fiber. The combined fields passes through an interference filter (IF) of 3 nm bandpass before entering the "NOON" state projection measurement. The output of the fiber coupler for the H-photons is mounted on a translation stage for the adjustment of the relative delay T_V between the H- and V-photons. The relative delay T_H between the two H-photons can be adjusted by another translation stage on the H-photon from BBO1. When $T_H >> T_c$ (T_c= the temporal width of the photons determined by the bandpass of the IF), the two H-photons are well separated and distinguishable but when $T_H << T_c$, the two H-photons becomes indistinguishable. The condition $T_H << T_c$ can be found by blocking the V-photon and observing a bump in two-photon coincidence of the two H-photons as we scan T_H (see the inset of Fig.3). Four-photon coincidence among ABCD detectors as well as two-photon coincidence between any two of the four detectors are measured. The four-photon coincidence is equivalent to the three-photon coincidence of ABC gated on the detection at D. The gated coincidence measurement ensures the two H-photons come from different crystals for the controllable distinguishability. The chance for two H-photons from the same crystal is rare and is from higher order case of three pairs.

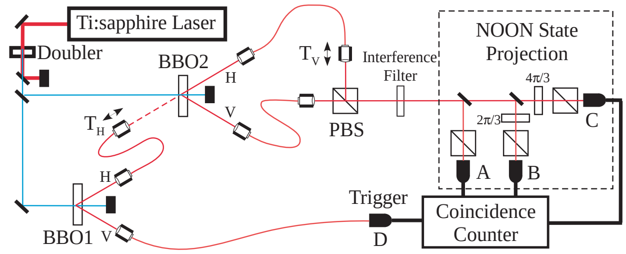

FIG. 2: *Setup for the experiment.*

The first experiment is performed when T_H is set at zero. We measure the four-photon coincidence among the ABCD detectors as we scan the delay T_V. The re-

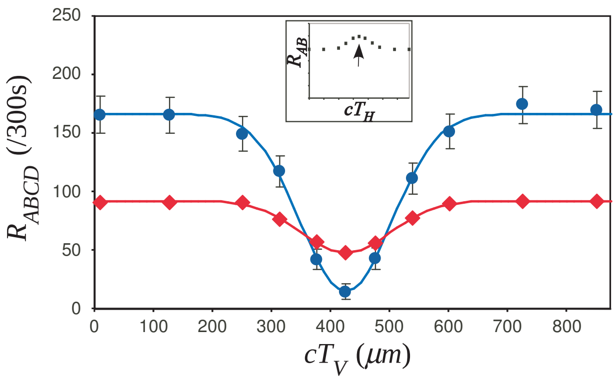

FIG. 3: *Four-photon coincidence as a function of the relative delay cT_V between the V-photon and the H-photons for the case of two overlapping H-photons ($T_H = 0$). Circles (blue): R_{ABCD}; Diamonds(red): $R_{ABCD}(2\times 2)$ derived from Eq.(11).*

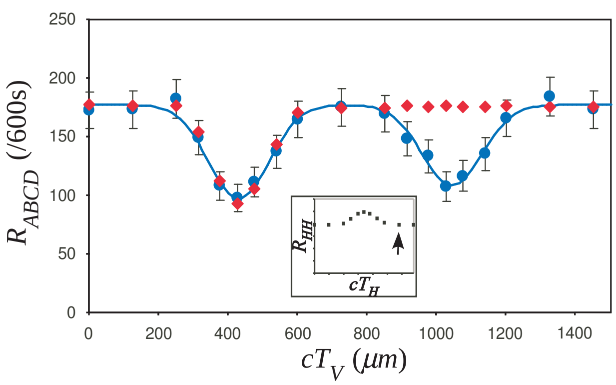

FIG. 4: *Four-photon coincidence as a function of the relative delay cT_V between the V-photon and the H-photons for two well separated H-photons ($T_H >> T_c$). Circles (blue): R_{ABCD}; Diamonds(red): $R_{ABCD}(2\times 2)$ derived from Eq.(11).*

sult of the scan after background subtraction is shown as the solid circles in Fig.3. The arrow in the inset shows the location of the delay T_H. The solid curve is a least square Gaussian fit with a visibility of $\mathcal{V}_3 = 91\%$ and a full width at half height (FWHH) of 185 μm. Next we set T_H far away from the peak of the bump as indicated by the arrow in the inset of Fig.4. We measure again the four-photon coincidence as a function of T_V. The background corrected data is shown in Fig.4 as the solid circles. The two interference dips are from the overlap of the V-photon with the two well separated H-photons, respectively. The solid curve is a least square double Gaussian fit with a FWHH = 200 μm and $\mathcal{V}_3 = 45\%, 39\%$, respectively. As expected from the prediction of Ref.[13], the single dip in Fig.3 with close to 100% visibility corresponds to the indistinguishable case whereas the double dips in Fig.4 with close to 50% visibility to the case of two distinguishable H-photons. The deviations from the exact predicted values are caused by the less than perfect situations to be discussed in the following.

There are two origins of imperfection. The first one is from spatial misalignment of all the fields. This is equivalent to spatial mode mismatch. In fact, this mode mismatch has been shown up in the two-photon coincidences (not plotted) between any two of the ABC detectors. Ideal two-photon interference in three-photon NOON state projection scheme should have 50% visibility but the observed two-photon visibility is 48% in the case of Fig.3 and is 46% in the case of Fig.4, which lead to a reduction factor of $\beta = 96\%, 92\%$, respectively.

The second cause is the temporal mode mismatch between the two pairs of photons generated from two crystals. This type of mode mismatch between different pairs of photons was encountered in numerous four-photon interference in parametric down-conversion. The best way to characterize this mismatch is by a quantity $\mathcal{E}/\mathcal{A}$ with $\mathcal{E}$ and $\mathcal{A}$ defined in Eqs.(2.15,2.16) of Ref.[20] and in general, we have $\mathcal{E} \le \mathcal{A}$. One extreme value of $\mathcal{E}/\mathcal{A} = 1$ corresponds to the situation when the two pairs from parametric down-conversion are completely overlapping in time and become indistinguishable four photons. The other extreme case of $\mathcal{E}/\mathcal{A} = 0$ is for the situation when the two pairs are well separated and become distinguishable. A simple application of the theory in Ref.[20] to the situation here gives that

$$\mathcal{V}_3(T_H << T_c) = \beta(\mathcal{A} + 3\mathcal{E})/2(\mathcal{A} + \mathcal{E}), \qquad (6)$$

$$\mathcal{V}_3^{(1)}(T_H >> T_c) = \beta/2, \qquad (7)$$

$$\mathcal{V}_3^{(2)}(T_H >> T_c) = \beta\mathcal{E}/2\mathcal{A}. \qquad (8)$$

From the measured values of β and $\mathcal{V}_3$, we may deduce the value of $\mathcal{E}/\mathcal{A}$ as

$$\mathcal{E}/\mathcal{A} = 0.82 \quad \text{from Eq.}(6), \qquad (9)$$

$$\mathcal{E}/\mathcal{A} = 0.86 \quad \text{from Eqs.}(7,8). \qquad (10)$$

Obviously, the two dips originate from the overlap of the V-photon with one of the two H-photons. To understand the difference in the visibility of the two dips, we plot in Fig.4 the four-photon coincidence (red diamond) deduced from two-photon coincidence by the formula

$$R_{ABCD}(2\times 2) = (R_{AB}R_{CD} + R_{AC}R_{BD} + R_{AD}R_{BC})/R_0, \qquad (11)$$

where R_0 is the repetition rate of the pump pulses. The label 2×2 indicates that Eq.(11) is based on the assumption that the two pairs are completely independent and the four-photon coincidence is purely accidental. The appearance of the lone dip in $R_{ABCD}(2\times 2)$ is due to the two-photon interference of the H and V photons from the same (second) crystal. The overlap of the first dip in R_{ABCD} with the lone dip in $R_{ABCD}(2\times 2)$ indicates that it is a two-photon interference effect. So the visibility of this dip does not depend on whether the two pairs are indistinguishable or not. The disappearance of the second dip in $R_{ABCD}(2\times 2)$ is consistent with Eq.(8) because

the 2×2 case leads to $\mathcal{E}/\mathcal{A} = 0$. So the dependence on $\mathcal{E}/\mathcal{A}$ for the visibility of the second dip in R_{ABCD} [Eq.(8)] indicates that the second dip is due to the overlap of the H-photon from the first crystal with the V-photon from the second crystal and the effect indeed requires the indistinguishability between the two pairs of photons, one from each crystal.

There is another independent way to measure the value of $\mathcal{E}/\mathcal{A}$, that is, to compare R_{ABCD} and $R_{ABCD}(2 \times 2)$ when $T_H = 0$ and $T_V = \pm\infty$. For this purpose, we plot $R_{ABCD}(2 \times 2)$ (red diamond) and the corresponding Gaussian fit (red solid curve) in Fig.3. By a similar analysis that leads to Eqs.(6-8), we find that the ratio $R_{ABCD}/R_{ABCD}(2 \times 2)$ at the wings of the scan in Fig.3 is $1 + \mathcal{E}/\mathcal{A}$. From the best fit values in Fig.3, we obtain $\mathcal{E}/\mathcal{A} = 0.81$. This value is consistent with the values in Eqs.(9,10) which are derived from the visibility in Fig.3 and 4. The visibility of 48% for the $R_{ABCD}(2 \times 2)$ curve in Fig.3 also leads to $\beta = 0.96$ for the $T_H = 0$ case, which is consistent with the two-photon data.

The multi-photon interferometric NOON state projection measurement scheme is a generalization of the well-known Hong-Ou-Mandel two-photon interferometer to arbitrary N-photon case. This statement is based on the facts that (1) this multi-photon interferometric scheme for $N = 2$ is exactly the Hong-Ou-Mandel two-photon interferometer and (2) the N-photon coincidence is always zero for any of the state $|k\rangle_H|N-k\rangle_V (k \neq 0, N)$.

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